Low-Order Models From FD-TD Time Samples

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***Abstract*—This letter introduces several techniques which allow one to create high-quality models of time domain signals produced by the finite-difference time-domain method. Guidelines for automated selection of key model parameters, such as the length of training sequence for model building and the model order, are given. The application of the criteria for extraction of time signature features based on the initial time sequences proposed in the paper is illustrated on two microwave filters.**

***Index Terms*—Finite difference-time domain (FD-TD), signal processing.**

I. Introduction

THE FINITE difference time domain (FD-TD) method is a versatile numerical technique that has been extensively used for solving various electromagnetic problems. One drawback of the method is a long computation time while modeling structures with high quality factors. In order to circumvent the problem signal processing techniques, such as all-pole auto-regressive (AR) [2], zero-pole auto-regressive moving-average (ARMA) [1], [2], and various variants of Prony's [3]–[6] methods, have been proposed to extract time signature features from a short segment of the original FD-TD sequence. Excellent results have been reported for most of these techniques, but each of them involves a nonobvious and subjective selection of key parameters. The successful application of all the aforementioned methods requires the knowledge of model parameters, namely the number of initial samples of the original FD-TD record to be discarded (the models should describe slowly decaying components of the signal), the decimation factor (due to the Courant limit the FD-TD method highly oversamples time sequences), and, finally, the order of the model. Despite many publications devoted to modeling of time domain signatures of microwave structures, except for [7], no general criteria for selecting models parameters have been defined and reported. As a result, in many cases, very high-order models had to be created [3], [4] for relatively simple filter structures.

This paper presents several techniques which can significantly facilitate the automated waveform feature extraction. The application of the criteria for extraction of time signature features based on the initial time sequences, proposed in the paper, is illustrated on two microwave filters.

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II. Signal Modeling Technique Used

There are many techniques which can be used for signal modeling [1]–[4], [6]. After a thorough examination of several methods the generalized pencil of function (GPOF) [6] has been found as a one providing the highest quality models. This technique builds a model of a signal in the form of superposition of P damped sinusoids

$$\hat{x}(n\Delta t) = \Re e\left\{\sum_{i=1}^{P} A_i e^{(\sigma_i + j\omega_i)n\Delta t + j\phi_i}\right\} \tag{1}$$

where A_i, σ_i, ω_i, and ϕ_i denote the real amplitude, damping factor, angular frequency and initial phase, respectively.

The method uses the singular value decomposition of the matrix spanning the signal space on singular vectors corresponding to the first P singular values. The damping factors and the angular frequencies are extracted from the eigenvalues of the P by P matrix constructed from left and right singular vectors. The amplitudes and initial phases are found by solving a linear least square problem after the preceding quantities have been obtained [6].

III. Criteria for Model Building

A. Selecting the Signal for Model Building

The time response of a microwave circuit consists of two parts: one dominated by transients and the other determining the late time behavior of the circuit's response. Since a model is used to describe slow decaying components of the signal, the first part is discarded and only the other part is used for model creation. To distinguish between these parts, the moving average value of the signal energy contained in a time window spanning at least a few periods of the signal is investigated. The moving average energy, expressed by the equation

$$E_l(n\Delta t) = \sum_{i=n-k}^{n} |x_l(i\Delta t)|^2 \tag{2}$$

where k is a window length, is calculated for each port (l) separately, during the FD-TD simulation.

The normalized value of moving average energy allows one to select the first and the last sample for modeling signals. This is illustrated in Fig. 1, which shows the moving average energy of the circuit in the logarithmic scale. The point -3 dB was chosen as a criterion for determining the first sample (n_1) for building a model. The last sample (n_2) was determined from the same auxiliary function by lowering the threshold to -16 dB. The threshold values are not restrictive and can be changed.

Application of the aforementioned approach guarantees that the sequence used for building the model is not contaminated

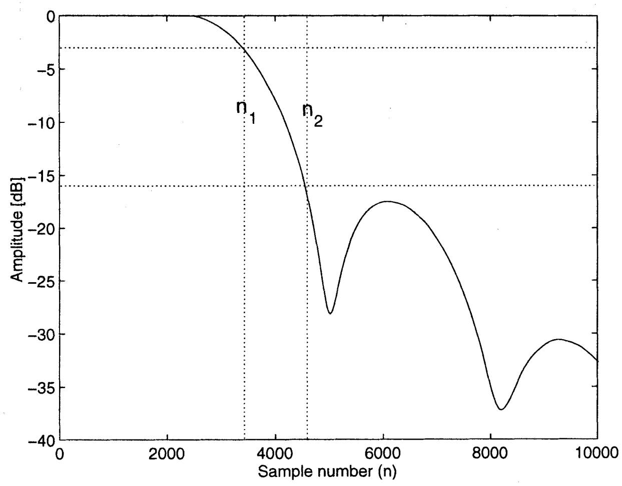

Fig. 1. Normalized average energy passing through the output port of the analyzed structure.

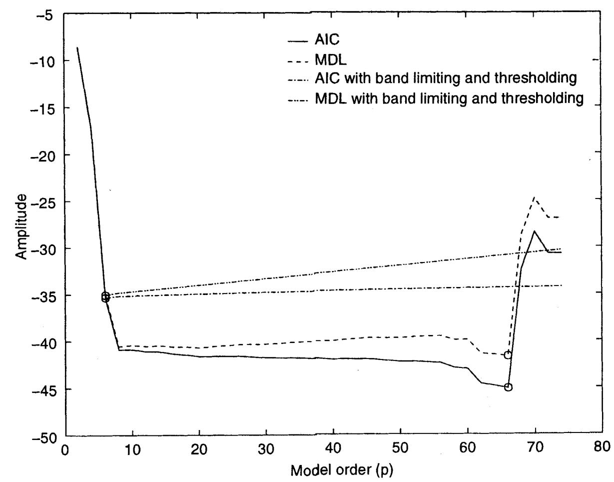

Fig. 2. Model order selection with and without band limiting and thresholding. The order selected is indicated by circles.

with the transient components and its length is adapted to the signal dynamics.

B. Desampling

Since the signals obtained from FD-TD method are highly oversampled, they cannot directly be used for model building. In order to select an appropriate decimation factor, the ratio of the Nyquist frequency to the upper frequency of the analyzed circuit is used. The expression can be written as

$$D = \frac{C}{2\,dt f_{\max}} \tag{3}$$

where dt is the time step used in the FD-TD modeling, $f_{\max}$ is the upper frequency of the analyzed structure, and C accounts for passing a signal through a low-pass digital filter in the decimation process. Numerical tests showed that the value of C equal to 0.75 provides the highest quality models.

C. Model Order Selection

One of the most important aspects of the use of signal modeling is a selection of model orders.

There are two commonly used statistics for selecting model orders [8], namely, the Akaike information criterion (AIC), which is based on selecting the order that minimizes

$$AIC(p) = \ln(\varepsilon^2) + 2p/((n_2 - n_1)/D) \tag{4}$$

and an information criterion selecting the order that minimizes the description length (MDL) defined as

$$MDL(p) = ((n_2 - n_1)/D)\ln(\varepsilon^2) + p\ln((n_2 - n_1)/D). \tag{5}$$

In both cases n_1, n_2, and D are defined in Sections III-A and III-B, and ε is a prediction error between the model and the decimated data samples

$$\varepsilon = \|\hat{x}(n\Delta t) - x(n\Delta t)\|. \tag{6}$$

Both of these statistics are used for model order selection simultaneously. The minimum of these measures is regarded as the best model order. Unfortunately, the criteria in some cases, may lead to contradicting results or to high model orders (Fig. 2). To overcome these problems, they have been modified as follows. All amplitudes of a model whose values are below 10^{-4} are set to zero, and so are the amplitudes of sinusoidal terms corresponding to the frequencies outside the band of interest. The first operation is called thresholding and the other is band-limiting. Both operations considerably improve the efficiency and consistency of model order selection (Fig. 2).

D. Frequency Response

The frequency responses of the investigated structures are calculated as a superposition of the discrete Fourier transform of the first part of the response, computed numerically

$$H_1(j\omega) = dt\sum_{n=0}^{n_2} x(n\,dt)e^{-j\omega n\,dt} \tag{7}$$

and an analytical formula expressed as

$$H_2(j\omega) = e^{-j\omega n_2\,dt}\left[\sum_{i=1}^{P}\frac{e^{(\sigma_i - j\omega_i)(n_2-n_1)\,dt}A_i e^{-j\phi_i}}{-\sigma_i + j(\omega_i+\omega)}\right]. \tag{8}$$

Thus, the frequency response is obtained after n_2 steps of time domain simulation. Generation of long time sequences and application of FFT are not necessary.

IV. Numerical Experiments

The techniques outlined in Section III were verified by modeling signals passing through the output port of two filters, namely, an H-plane three-cavity filter [9] and a dual-mode cylindrical cavity filter [10]. The commercial QuickWave-3-D [11] software was used to provide the FD-TD data. Table I summarizes the results. The error norms given in the last row of the table were calculated in the frequency domain with reference to the Fourier transforms of very long time sequences obtained from direct FD-TD simulations. In both cases, the number of complex conjugated pairs of poles, which equals half of the model order P, coincides with the number of their electrical resonators.

TABLE I
RESULTS OF MODELING

Filter	1	2
Frequency range [GHz]	10-15	14.5-15.5
Desampling factor	27	30
First/last (n_1/n_2) sample	3429/4590	2100/10770
Mode order	6	4
Error norm with model	$6.4 \cdot 10^{-7}$	$8.2 \cdot 10^{-6}$

1. H-plane three-cavity filter [9]
2. Dual-mode cylindrical cavity filter [10]

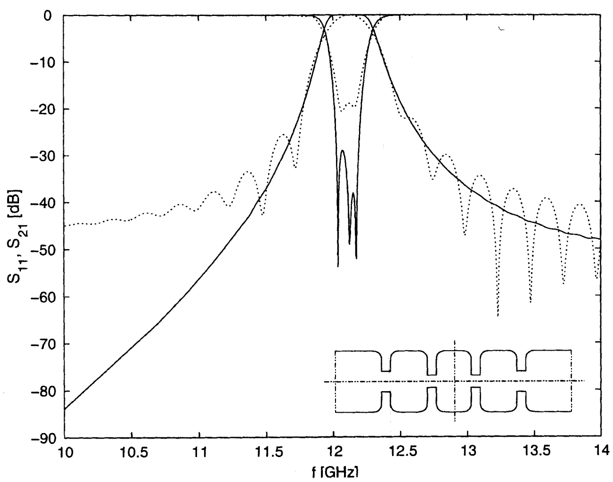

Fig. 3. Scattering parameters (S_{11}, S_{21}) of the H-plane three-cavity filter obtained by modeling (—) and Fourier transform of a short (n_2-length) time response ($\cdots$).

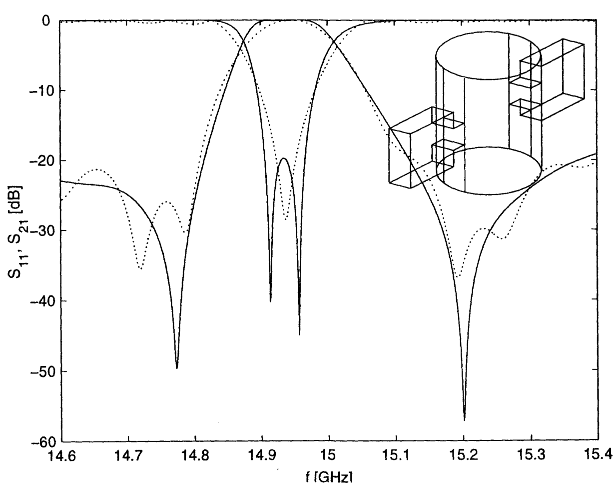

Fig. 4. Scattering parameters (S_{11}, S_{21}) of the dual-mode cylindrical cavity filter obtained by modeling (—) and Fourier transform of a short (n_2-length) time response ($\cdots$).

To illustrate the improvement which is gained by the application of the automatically created models, the filter characteristics obtained from the short time record with and without modeling are shown in Figs. 3 and 4. The dotted lines correspond to the Fourier transform alone, computed using (7) and a short n_2-length time sequence. The solid lines correspond to the frequency response of the filters obtained by combining the FD-TD algorithm with the models as described in Section III-D.

V. CONLCUSIONS

Using a few simple techniques, it is now possible to develop efficient procedures with automatic selection of time sequence for model building and order selection suitable for modern CAD packages. The best solution which emerges from the above discussion is to use GPOF method together with AIC and MDL statistics with band limiting and thresholding built-in for model order selection and normalized moving average energy for determining the training sequence. The model-building procedures described in this letter have been found to be very effective in optimization of microwave filter structures using commercial time domain CAD software [12].

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